An Accurate Measurement Technique for Line Properties, Junction Effects, and Dielectric and Magnetic Material Parameters

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Abstract —**Transmission/reflection coefficients of unknown transmission lines are analyzed. The characteristic impedance, the propagation constant, and the parameters of the junctions at the connections with the measurement setup can be calculated if the coefficients of three different lengths of the line being investigated are measured. Therefore it is called the L^3 method (line/line/line). Transmission data suffice for the determination of only the propagation constant. They are used in the case of material parameter measurements with loaded lines. Dielectric and magnetic properties of the filling material are calculated via the set of transverse resonance equations. Even nonreciprocal lines and off-diagonal tensor elements are amenable as are the properties of fluids or powders. Due to the availability of an analytically exact formulation, the overall accuracy achievable can be precisely specified, e.g. 0.1 percent in the X-band for the absolute values of material parameters. The accuracy is dependent mainly on geometrical tolerances if precise vector network analyzers are used.**

I. Introduction

Transmission line properties and junction effects between different lines are essential features in the whole field of microwaves. Though sometimes they can be calculated analytically with greater or lesser expense and accuracy, there often is a need for measurement. This is especially true for the determination of dielectric and magnetic material properties, which by definition is done by measuring their influence on the propagation properties of electromagnetic waves.

A particular problem concerning such measurements at microwave frequencies has to do with the scattering processes at junctions of two transmission lines having different properties, which must explicitly be considered in the description of wave propagation. In Section II it will be shown that the measurement of three different lengths $l^{(m)}$ ($m = 1,2,3$) of an unknown line is sufficient to determine all properties of the unknown line and of the junctions to the measurement lines.

In Section III the evaluation of dielectric and magnetic material parameters is given. As a specific example slabs are placed in a rectangular waveguide in special loading configurations. The propagation constants are determined with the method described in Section II. The simultaneous solution of the set of transverse resonance equations yields the unknown values. Not all possible cases can be thoroughly dealt with in this publication, but an altered procedure does not give rise to additional difficulties due to the universality of the method. However, a ferrite-slab-loaded rectangular waveguide is of importance for the construction of nonreciprocal devices. Therefore in Section IV a method for obtaining the ferrite's off-diagonal magnetic tensor elements is outlined.

In Section V the accuracy and some practical hints are dealt with, while in Section VI experimental results are discussed.

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II. Analysis of Transmission/Reflection Coefficients

An electromagnetic wave induces surface and/or displacement currents at a geometrical discontinuity (e.g. a junction) in a transmission line. If the wavelength is large compared to the discontinuity, the fields may be regarded as spatially stationary. Then the normal low-frequency transmission/reflection behavior is given by the lines' impedance ratio, and the special geometry has no further influence. At microwave frequencies, however, the discontinuity currents give rise to a considerable amount of energy flow in field distortions, which alter the propagation behavior. The distortions can, for example, be described in terms of an expansion in a suitable set of orthogonal higher modes. Generally speaking, Fresnel's formulas must be replaced by a geometry-dependent scattering formulation.

In order to have well-defined field distributions along a microwave line, usual line geometries just allow the propagation of the lowest, or fundamental, mode. Then the excited higher modes at a junction decay exponentially with the distance to the junction; i.e., their propagation constant is mainly imaginary. In this case the propagation of the fundamental mode may be described in terms of conventional two-port and transmission line theory, and a

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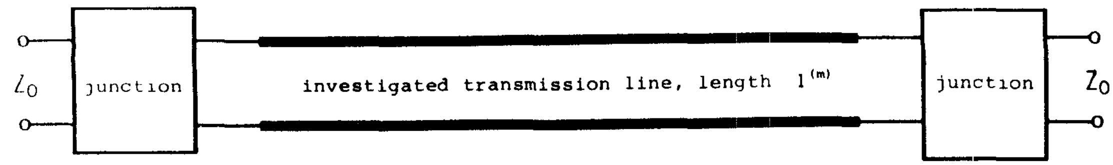

(a)

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{reciprocal} \quad \begin{pmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{pmatrix}$$

$$\begin{pmatrix} \cosh \gamma l^{(m)} & Z_p \sinh \gamma l^{(m)} \\ Z_p^{-1} \sinh \gamma l^{(m)} & \cosh \gamma l^{(m)} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{non-reciprocal} \quad \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{pmatrix}$$

$$\frac{e^{(\gamma_1 - \gamma_2)l^{(m)}}}{Z_{p1}^{-1} + Z_{p2}^{-1}} \begin{pmatrix} Z_{p2}^{-1} e^{\gamma_2 l^{(m)}} + Z_{p1}^{-1} e^{-\gamma_1 l^{(m)}} & e^{\gamma_2 l^{(m)}} - e^{-\gamma_1 l^{(m)}} \\ (Z_{p1} Z_{p2})^{-1} (e^{\gamma_2 l^{(m)}} - e^{-\gamma_1 l^{(m)}}) & Z_{p1}^{-1} e^{\gamma_2 l^{(m)}} + Z_{p2}^{-1} e^{-\gamma_1 l^{(m)}} \end{pmatrix}$$

(b)

Fig. 1. (a) Equivalent circuit and (b) corresponding chain matrices of an unknown transmission line inserted in a measuring line.

junction can be regarded as a passive, linear two-port. This is a much easier description than the theoretical consideration of the explicit field distributions, but demands an exact measurement method for the defining parameters.

The concept now is to calculate the transmission/reflection coefficients as a function of all these parameters. The formulas obtained will specify the necessary measurements.

The equivalent circuit of an unknown line having a length $l^{(m)}$ inserted in a measurement line may be sketched as in Fig. 1. The following notation is chosen, all values being potentially complex:

$a_{11}, a_{12}, a_{21}, a_{22}$	chain matrix parameters of the junction;
Z_0, Z_p	characteristic impedances of measurement and unknown line;
γ	propagation constant of unknown line;
$T_{\text{meas}}^{(m)}, T_{\text{calc}}^{(m)}$ $\Gamma_{\text{meas}}^{(m)}, \Gamma_{\text{calc}}^{(m)}$	measured and calculated transmission/reflection coefficients.

There are two assumptions which must be fulfilled to apply this equivalent circuit:

a) the unknown line must be uniform in impedance and propagation constant along the propagation direction;
b) only the fundamental mode propagates in the measurement setup, i.e., the decay of the higher modes must be short in comparison with $l^{(m)}$.

While a) is obvious, the treatment of b) will be explained in Section III.

The chain matrix is the usual one connecting the input voltage/current with the output values. Voltage and current are proportional to the electric and magnetic field strengths, impedance being their ratio, as has been explained in an early work [1]. The more common notation of the wave cascading matrix [2] would equivalently be possible, but does not give direct insight into the ratio Z_p/Z_0. It should, however, be emphasized that only this ratio determines the propagation properties. Z_0 can have any predefined value and Z_p is measured relative to it. The actual measured values, the transmission/reflection coefficients given here and the common scattering parameters (S parameters), are of course identical.

According to network theory, the two-port parameters of the entire equivalent circuit of Fig. 1 are given by successive multiplication of the individual matrices in the direction to the source. From this product matrix the calculated transmission/reflection coefficients are obtained. The simultaneous solution of the following equations ($m = 1, 2, 3$) yields the unknown parameters:

$$T_{\text{meas}}^{(m)} - T_{\text{calc}}^{(m)} = 0$$
$$\Gamma_{\text{meas}}^{(m)} - \Gamma_{\text{calc}}^{(m)} = 0. \tag{1}$$

In the case of a reciprocal line the explicit formulas are the same for the forward/reversed directions:

$$T_{\text{calc}}^{(m)} = \frac{2}{A \cosh[\gamma l^{(m)}] + B \sinh[\gamma l^{(m)}]}$$
$$\Gamma_{\text{calc}}^{(m)} = \frac{C \cosh[\gamma l^{(m)}] + D \sinh[\gamma l^{(m)}]}{A \cosh[\gamma l^{(m)}] + B \sinh[\gamma l^{(m)}]}. \tag{2}$$

A, B, C, and D are given by

$$A = 2\left(a_{11}a_{22} + a_{12}a_{21} + Z_0 a_{21}a_{22} + \frac{1}{Z_0}a_{11}a_{12}\right)$$

$$B = \frac{Z_p}{Z_0}a_{11}^2 + \frac{1}{Z_0 Z_p}a_{12}^2 + Z_0 Z_p a_{21}^2 + \frac{Z_p}{Z_0}a_{22}^2 + 2Z_p a_{11}a_{21} + \frac{2}{Z_p}a_{12}a_{22}$$

$$C = 2\left(\frac{1}{Z_0}a_{11}a_{12} - Z_0 a_{21}a_{22}\right)$$

$$D = \frac{Z_p}{Z_0}a_{11}^2 + \frac{1}{Z_0 Z_p}a_{12}^2 - Z_0 Z_p a_{21}^2 - \frac{Z_0}{Z_p}a_{22}^2. \quad (3)$$

Here γ, Z_p and only three of the four parameters a_{ij} are unknown, since $a_{11}a_{22} - a_{12}a_{21} = 1$ (reciprocity). It can be seen that in any case, the values $\mathsf{T}_{\text{meas}}, \Gamma_{\text{meas}}$ of three different lengths $l^{(m)}$ of the investigated transmission line completely determine the five unknown parameters A, B, C, D, and γ. The specification of the lengths must be the absolute geometrical one if the a_{ij} of the junctions themselves are to be determined. If only the differences in geometrical lengths are specified for the $l^{(m)}$, i.e., the same offset Δl is added to or subtracted from each absolute geometrical length, the line properties of Δl are calculated as additional junction effects (changes in a_{ij}), but γ and Z_p do not change. This is a consequence of the "in-line calibration" properties of the method. If the geometrical discontinuities of the junction extend over a certain region in the propagation direction, the a_{ij} of this region are calculated by specifying the "inner" lengths $l^{(m)}$ corresponding to the uniform line sections.

It can also be seen that if the propagation constant alone is of interest (e.g. for the determination of material parameters), then according to (2) the transmission coefficients suffice for the calculation of A, B, and γ. The in and out junctions may be different, e.g. have different geometries, because they only influence the values A and B, which are not interpreted further. In this case, an implicit calibration of the measurement setup can be done (for further information on calibration and error two-ports, see [3] and [4]). Consider samples which are successively inserted in a waveguide. If the face directed to the source is always at the same place, the source-related error two-port combined with the input junction two-port is a constant during the successive measurements, which automatically removes the influence of these errors on the calculated propagation constant. This was done during the measurements presented in Section VI.

The solution of (1)–(3) has several aspects. First (1) and (2) must be solved. They are explicit for the values A, B, C, and D and implicit for γ. Consequently the one-dimensional complex Newton iteration can be used, which does not require great numerical effort. The junction parameters and the line's impedance are calculated by (3) after specifying a value for Z_0. More than three lengths ($m = 1, 2, 3, \cdots$) can be used to find the best values by averaging. In this case the explicit method of least squares must be used by searching for those values A, B, C, D, and γ for which the sum of the square distances between measured and calculated T and Γ coefficients in the complex plane has a minimum. Though this requires a little more effort, the necessary matrix routines are available on nearly every computer. It takes about 15 seconds on an IBM AT with numerical coprocessor to find the best solutions A, B, and γ for the transmission data of six different lengths (double precision). This procedure was used in Section VI.

At first sight the method given here seems to be similar to an advanced calibration method for network analyzers, the line/reflect/line (LRL) method [3]. This method was developed to determine the repeatable measurement errors on both sides of a two-port device under test (DUT). In fact the LRL method can be understood by the formalism given here if the junctions on both sides of the line are regarded as error two-ports. But an essential difference is that the error two-ports on both sides are *not* equal. The consequence can be understood in the following way: Weissfloch was the first to show [5] that a passive, linear two-port can be most naturally thought of as being composed of a left and a right extension of the feeding transmission lines, separated by an ideal transformer. A calibration technique using lines can therefore determine the sum of electrical length change of a line under test, but not the individual changes on the left and the right. Consequently single reflection measurements at both measurement ports with the same reflection standard are compelling for achieving this separation, as is done using LRL. Here, however, having *a priori* equal junctions the full information can be obtained *only* using lines, as is the case if the two junctions are different, and only the propagation constant must be determined. This is of advantage if the realization of a reproducible, interchangeable reflective standard in the investigated line is impractical or even impossible. Furthermore the LRL method is not able to separate the impedance ratio Z_p/Z_0 from the junction parameters. Also there are differences in the determination of propagation constants: LRL uses transmission/reflection data of two line lengths for this task, whereas only transmission data of three are used here. The reflection coefficient of usable line geometries for material parameter measurements (see Section III) is low (often < 0.05), resulting in a low measurement accuracy of reflection data. Furthermore the absolute geometrical positions of the samples in the waveguide must be known. These disadvantages of LRL are avoided using the method of this paper, which results in a higher accuracy and the possibility of determining low-loss factors. Of course the LRL method was developed for calibration and not for the purposes described here.

It is suggested that the method presented here be referred to as the L^3 method (line/line/line).

III. The Measurement of Material Properties

The transverse resonance equation connects the propagation constant of a transmission line with its transverse structure, i.e., the transverse boundary conditions and the properties of the filling (loading) materials [6], [7]. This article does not aim to give a detailed review of the various published material parameter measurements based on this principle (mostly in waveguide technique). Only some of them consider the junction effects by evaluating the higher mode distribution, which entails a complicated self-consistent computation. They are not universal. For all others the following considerations prove correct:

a) If the waveguide is completely filled with the tested material, the junction effects are small but still present due to the nonexistence of ideal geometry. At least the next higher mode is able to propagate in the loaded region, so that the fundamental mode evaluation is no longer appropriate.
b) If a partially filled waveguide is used, the sample geometry can be chosen due to the supposed material parameters to achieve an optimal response. Furthermore a higher mode propagation is suppressed in a certain frequency region. However, the junction effects are neglected, at least in part.

A reduction in accuracy is the consequence of a) and b), which can hardly be desirable. For example there is ambiguity in assigning an attenuation of measured transmission data to sample reflections or to "real" material losses, which cannot be compensated by the additional measurement of less precise reflection data. The determination of low loss factors has therefore been restricted to resonator methods. Of course, resonators have the definite disadvantage in yielding no frequency information. Furthermore, most of them work with only approximative formulas. An example is the hole in many resonators necessary for inserting the rod-like sample. The hole's electromagnetic properties depend on the unknown sample parameters and can at most be considered by gauge substances. Therefore the accuracies in technical data sheets of microwave materials are mostly related to the accuracy of the measurement equipment and not to the methodical, systematic errors. Using the L^3 method, all these shortcomings are removed.

The propagation constants obtained are now used as input for the simultaneous solution of the corresponding transverse resonance equations. Different loading configurations and/or line types must be used, as will become clear with the given examples. This concept is universal and will now be demonstrated with the help of a H_{10} mode rectangular waveguide. Two loading configurations are shown in Fig. 2; the notation is as follows (all values are again potentially complex except the geometrical ones):

a, d_1, d_2	dimensions of Fig. 2,
λ_{vac}	vacuum wavelength of measurement frequency,
$\epsilon = \epsilon' - i\epsilon''$	dielectric constant (a missing index denotes isotropy),
$\mu = \mu' - i\mu''$	magnetic permeability (a missing index denotes isotropy).

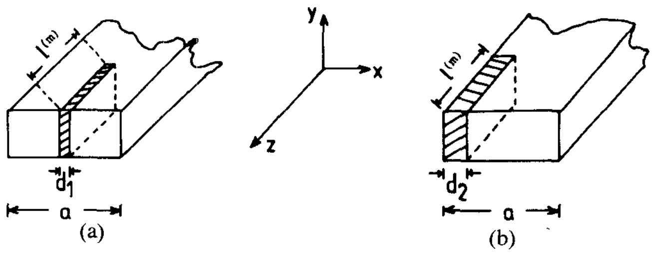

Fig. 2. Loading configurations in a rectangular waveguide.

The ratios of the imaginary and real parts define the so-called loss tangents of the material parameters. If the off-diagonal tensor elements of the material parameters are zero, the complex transverse resonance equations for the material orientations according to Fig. 2(a) and (b) are ($\gamma^{(a)}, \gamma^{(b)}$ being the two propagation constants)

$$\frac{1}{\sqrt{1-n^2}}\tan\left[\frac{(a-d_1)\pi}{k\cdot\lambda_{vac}}\sqrt{1-n^2}\right] = \frac{\mu_z}{\sqrt{\epsilon_y\mu_z-\frac{\mu_z}{\mu_x}n^2}}\cot\left[\frac{d_1\pi}{k\cdot\lambda_{vac}}\sqrt{\epsilon_y\mu_z-\frac{\mu_z}{\mu_x}n^2}\right] \qquad (4a)$$

$$n = \frac{\lambda_{vac}\cdot\gamma^{(a,b)}}{i\cdot 2\pi}$$

$$\frac{\mu_z}{\sqrt{\epsilon_y\mu_z-\frac{\mu_z}{\mu_x}n^2}}\tan\left[\frac{2\pi d_2}{k\cdot\lambda_{vac}}\sqrt{\epsilon_y\mu_z-\frac{\mu_z}{\mu_x}n^2}\right] = -\frac{1}{\sqrt{1-n^2}}\tan\left[\frac{2\pi(a-d_2)}{k\cdot\lambda_{vac}}\sqrt{1-n^2}\right]. \qquad (4b)$$

The factor k denotes the H_{k0}-mode type, and these modes are the only ones to be excited at the junction for geometrical reasons. First the decay of the excited modes shall be dealt with to ensure that the equivalent circuit of Fig. 1 can be applied (see Section II). The H_{20} mode has the lowest attenuation of all higher modes. If the highest supposed values for the material constants are inserted in (4), the H_{20} propagation constant can be calculated. It should be mainly imaginary to yield a high attenuation. "Mainly" can be made more precise by the following guideline: the H_{20} fields should decay to 5 percent of an initial maximum on the length of the shortest sample. Then only 0.25 percent of the inital energy at one junction reaches the other sample end, and since the total junction effect on transmission is normally below 15 percent in phase, this will suffice. Due to this or possibly other conditions, the usable frequency range for a given geometry or vice versa can be determined. The further higher modes are attenuated more and more and significantly surpass this condition. Also, an experimental procedure for testing the geometry can be performed by simply measuring a series of shorter and shorter samples. If the results between the triplets of the lengths become inconsistent, they are too short in comparison to the decay of the higher modes. With some experience appropriate conditions can be chosen for the "safe range" without carrying out these procedures. There is, however, a puzzling thing with the side position (Fig. 2(b) and eq. (4b)), which is a result of the asymmetry. Already in the case $k=1$ there can be two different solutions for the propagation constant. However, their cutoff frequencies are still separated, the lower defining the fundamental mode. So the given treatment must be applied for the higher of these two modes.

Next eqs. (4) are analyzed for the measured propagation constant of the fundamental mode ($k=1$). For purely dielectric materials ($\mu=1$), a maximal response is obtained in the middle of the waveguide due to the electric field maximum (Fig. 2(a)). Having differently oriented samples, all diagonal elements of the dielectric tensor can be determined. Each value is given by the solution of the implicit eq. (4a), which can be done via the one-dimensional complex Newton procedure. In the side position, the magnetic properties in the z direction mostly influence the propagation. An isotropic permeability μ and one of the ϵ_j are obtained by measuring $\gamma^{(a)}$, $\gamma^{(b)}$ and by the simultaneous solution of the implicit equations (4a) and (4b). This requires a two-dimensional complex Newton procedure.

It should be mentioned that the magnetic properties are not amenable to the simultaneous measurement of transmission/reflection in one of the positions. According to Section II, the impedance Z_p can be evaluated, but there is no formula connecting Z_p with the magnetic properties.

Both μ_x and μ_z can be obtained having two sets of samples with changed directions of the x and z material orientation. The middle and side positions must be measured for one of the sets but only the side position for the other. The three resultant propagation constants are needed for the simultaneous solution of one equation from (4a) and two from (4b) (three-dimensional complex Newton procedure). Keeping this formalism in mind, it is clear that

a) for each position the samples' thicknesses and lengths may alter (but must not) to be in a range where measured transmission data have the highest accuracy;
b) also the waveguide or fundamental mode type may differ in the same set of simultaneous equations;
c) all complex diagonal dielectric and magnetic tensor elements can be obtained.

Viscous and powdery materials need a boundary to be measured, i.e., sidewalls in the form of a container giving a well-defined rectangular cross section. Three of them with different lengths and identical cross sections are needed. According to Section II, the front and end walls are junction effects. For simplicity, $\mu=1$ and the symmetric arrangement of Fig. 2(a) (ϵ_y measurement) was chosen with equal container walls (thickness d_c) on both sides of the sample, but a general treatment is possible. The transverse resonance equation is given below for this sandwich-like structure. The left-hand side of this equation is given by the successive transverse impedance transformation of the waveguide wall impedance (short) to the outer container wall and then to the inner container wall. The (complex) dielectric constant ϵ_c of the container material must be known. Still the evaluation is analytically exact. The left-hand side of (5) is constant during iteration, so the solution is as quickly found as the one for eqs. (4) (about 0.3 seconds on the IBM/AT with coprocessor):

$$\frac{\tan\left[\frac{\pi}{\lambda_{\text{vac}}}(a-d_1-2d_c)\sqrt{1-n^2}\right]+\frac{\sqrt{1-n^2}}{\sqrt{\epsilon_c-n^2}}\tan\left[\frac{d_c\cdot 2\pi}{\lambda_{\text{vac}}}\sqrt{\epsilon_c-n^2}\right]}{\sqrt{1-n^2}-\sqrt{\epsilon_c-n^2}\tan\left[\frac{\pi}{\lambda_{\text{vac}}}(a-d_1-2d_c)\sqrt{1-n^2}\right]\tan\left[\frac{d_c\cdot 2\pi}{\lambda_{\text{vac}}}\sqrt{\epsilon_c-n^2}\right]}=\frac{1}{\sqrt{\epsilon_y-n^2}}\cot\left[\frac{d_1\cdot\pi}{\lambda_{\text{vac}}}\sqrt{\epsilon_y-n^2}\right] \tag{5}$$

$$n=\frac{\lambda_{\text{vac}}\cdot\gamma^{(a)}}{i\cdot 2\pi}.$$

If the top and bottom sides of the container are sealed with foils for practical reasons, the foils should be as thin as possible. A rigorous treatment is no longer possible. A correction calculation can be made according to Section V, if the dielectric constant of the foil is known. There are foils of 2 μm thickness, which can be heat-sealed on plastic containers and allow this correction to be made up to absolute values of the dielectric constants as high as 100 in the X-band (8.2–12.4 GHz).

The transverse resonance equation for an asymmetric sandwich layer (e.g. an epitaxial layer on a substrate) can be found in a way similar to that for the sandwich-like structure.

IV. Off-Diagonal Tensor Properties

The procedure is sketched briefly. The example is a ferrite slab loading in the form of Fig. 2. The ferrite is magnetized in the y direction, leading to a magnetic susceptibility tensor with four elements [8]. A nonreciprocity of the line is a consequence with different forward/reverse propagation constants γ_1, γ_2 and impedances Z_{p1}, Z_{p2}. The transmission/reflection coefficients are found in the forward direction by applying the nonreciprocal matrices of Fig. 1, for doing this in the reverse direction all indices of Z and γ in the line matrix have to be interchanged. Data for both directions must be measured; the set of equations (1)–(3) is therefore expanded by a factor of two. The cosh and sinh functions of (2) are replaced by the four terms $e^{\gamma_1 l^{(m)}}$, $e^{-\gamma_1 l^{(m)}}$ $e^{\gamma_2 l^{(m)}}$, and $e^{-\gamma_2 l^{(m)}}$ and corresponding factors. The whole set can be solved to obtain all unknown parameters; again transmission data suffice for the determination of γ_1, γ_2 and the material parameters.

The adequate transverse resonance equations are often dealt with in the literature for the lossy case as well [9]. They are different for the forward/reverse directions involving the tensor components in different ways. Consequently the off-diagonal components can also be determined by their simultaneous solution. A reverse wave may be below cutoff having a high attenuation: as long as its transmission response can be measured, the whole procedure may be used again.

Off-diagonal dielectric tensor elements can be determined in an analogous manner using an E-mode driven waveguide.

V. Accuracy Considerations and Practical Hints

All uncertainties of measured values and input parameters can be considered with regard to their influence on the overall accuracy of determined parameters, since the formulas give the analytical relationship. This could be done in a systematic way with a Monte Carlo analysis, but is not necessary and was performed only in part to obtain the essential influences.

The achievable accuracies for two-port parameters and propagation constants have been impressively demonstrated for the LRL method [10]. Considering the technical data of modern vector network analyzers and calibrating them with LRL, an accuracy of better than 0.1 percent is possible in any case for the determined parameters of Section II. In the case of material parameter measurements the more precise transmission data are used. A further accuracy enhancement is achieved by a second implicit calibration during the measurement as described in Section II. According to the Monte Carlo analysis, the accuracies of transmission data achieved are no longer important for material parameter measurements. The residual limitations are determined only by the tolerances of geometry and of material homogeneity.

Insufficient homogeneity is an intractable problem in every integrating method and therefore is assumed to be negligible in all such measurements. But L^3 is a good test for homogeneity in the case of low-loss substances: if the three lengths of the sample induce slightly different line properties, the formulas can be shown to simulate a loss factor that is too high, sometimes even with wrong sign. This is quite remarkable, because very small sample volumes can be tested for uniformity by using a rectangular waveguide with reduced height.

The geometrical tolerances must be treated more elaborately for each geometry used. In the following the geometry of Fig. 2 is dealt with. One factor is the occurrence of clearance between the sample and the upper waveguide wall (air gap). At first sight this is a serious problem, because it cannot be analytically handled. Experiments were carried out by increasing the air gap systematically with different sample sets of dielectrics. Corrections can be evaluated by regarding the arrangement as an air gap or a foil capacitor and a sample capacitor, yielding the measured dielectric constant of the series capacitor having the full height. This can be done in the lossy case, too. It works up to corrections of 5 percent for the absolute values of the material parameters and should be applied using thin foils as described in Section III for the measurement of fluids. However, if the dielectric constant of the material is high and/or a high accuracy is desired, the air gap should for the most part be avoided. This can be achieved by the precise construction of a waveguide with a removable cover that is fixed by a strong clamping device. Then the samples can be exactly positioned from the top with micrometer adjustments in both directions and can be higher by a few μm than the waveguide itself. The deformation (change of geometry) by quenching a sample this small amount is negligibly low due to elasticity, and the air gap is no longer a problem. Best results will be achieved if the samples' wall-contacting faces are metallized or by using a conductive paste.

The remaining geometrical tolerances belong to the specification of the sample lengths, thicknesses, and positions in the waveguide and are actually the important parameters. In varying their values as the input parameters of the whole method, the influence in actual measurement situations was analyzed. The technically achievable tolerances are about ± 1 μm. Then the overall accuracy of the absolute values of the material parameters is about 0.1 percent in the X-band (8.2–12.4 GHz) under the conditions given in Section VI. The accuracy of the loss tangent for low-loss substances is 10^{-6} to 10^{-7}. At higher frequencies the values increase in proportion to the decrease in dimensions due to constant geometrical tolerances. A tolerance of ± 10 μm is always sufficient for a 1 percent accuracy of the absolute values at 10 GHz, and in special situations (thick sample, low parameter values) ± 25 μm will suffice for this. The specific geometry of Fig. 2 seems to be useful up to 300 GHz if adequate samples can be prepared and handled.

Measuring at high frequencies (>50 GHz) it is also possible to consider the nonideal conductivity of the waveguide walls. Then the wall impedance in the transverse resonance circuit is not a short; rather, it is proportional to the inverse product of conductivity and skin depth [11].

VI. Experimental Results for Material Parameters

The results shown in Table I have been obtained on an experimental setup which is not sufficiently modern to achieve a high absolute accuracy. It has "only" a superior relative resolution in detecting changes in dielectric constant as a function of temperature ($-180°$ to $+500°$C), reaching values of 10^{-3}. However, the value of the L^3 method with implicit calibration can also be seen on this setup. It is an X-band microwave bridge consisting of a HP phase shifter 885A (accuracy $\pm 3°$) and a HP attenuator 382A (accuracy ± 0.3 dB) in the reference path, both being mechanically tuned (for the temperature measurements this is done by an electronically controlled zero set with motor drives). Due to the mechanical tuning a

TABLE I
EXPERIMENTAL RESULTS

material (manufacturer)	real part of dielectric constant: data sheet	real part of dielectric constant: with ±accuracies	resolution of dielectric loss factor $\tan\delta_\varepsilon$ (see text) $\times 10^{-2}$	real part of magnetic permeability with ±accuracies	resolution of magnetic loss factor $\tan\delta_\mu$ (see text) $\times 10^{-2}$	measurement frequency in GHz	different lengths measured
Teflon PFA 340 (Du Pont)	2.1	2.06 2%	0.2	-	-	8.492	8
"	"	2.04 2.5%	2	1.02 3.5%	5	"	"
"	"	2.07 2%	0.35	-	-	10.280	"
DiClad 522 (Keene)	2.50	2.91 3%	2	-	-	9.429	7
DiClad 810 (Keene)	10.50	12.2 4%	1.5	-	-	"	"
RT/duroid 5880 (Rogers)	2.20	2.19 2%	2.5	-	-	"	"
RT/duroid 6006 (Rogers)	6.00	7.42 3.5%	1	-	-	"	"
RT/duroid 6010.5(Rogers)	10.50	12.2 4%	1.5	-	-	"	"
TLY-5-0930 (Taconic)	2.18	2.29 2.5%	2	-	-	"	"
TLX-9-1250 (Taconic)	2.48	2.92 2.5%	2.5	-	-	"	"
DMAT - 9.8 (Trans-Tech)	9.96	11.2 4%	2	-	-	8.471	6
D 8623 (Trans-Tech)	75.0	73 10%	0.3	-	-	"	5
99.5% BeO (Consol.	6.60	6.73 2.5%	0.2	-	-	8.583	10
Beryllium)	"	6.88 3.5%	2	0.96 4%	3.5	"	"
high purity silicon,	-	11.4 4%	0.25	-	-	8.585	5
single crystal (Wacker)	-	11.2 5%	0.2	1.04 5%	0.1	"	"

broad-band measurement is not possible. The two 3 dB directional couplers have directivities of 30 dB. Three unilines with isolations of 30 dB suppress standing waves. The measuring path has a waveguide with a removable cover and a 5 μm hard gold coating. Geometries as presented in Fig. 2 were used.

Positioning was manually controlled using a stereo microscope (achievable tolerance ± 100 μm); the largest tolerances of the samples were about ± 10 μm for the hard materials and ± 25 μm for the softer ones. The laminates' copper had been removed in a $FeCl_3$ etching bath. The samples' lengths ranged from 12 to 40 mm, and the thicknesses varied between 0.3 and 5 mm, according to the supposed parameters. The measurement accuracies of the table were obtained by the methods of Section V, including the uncertainties of the transmission data, of course. The given data of the laminates are the in-sheet-plane values, but all measured materials are supposed to be isotropic in dielectric and magnetic behavior. Measurements were made at room temperature.

The data prove that even with this unsophisticated equipment the L^3 method gives usable results. They yield a better overall accuracy than most of the industry standards. The magnetic permeability was also measured in some cases; the evaluation is given first only for the dielectric constant ($\mu = 1$) and then for both parameters. The accuracy of the side position measurement is worse than the middle one, since the absolute change in transmission is low for pure dielectrics and should have been optimized in geometry (different sample set), which was not done. Sample D8623 had a small air gap to the waveguide cover, resulting in a low accuracy.

The effect of the junctions themselves can be described as a shortening of the electrical length of the line, as is consistent with theoretical investigations [12]. In comparison with a "normal" transmission line evaluation (which means $a_{11} = a_{22} = 1$, $a_{12} = a_{21} = 0$) the real part of the dielectric constant is raised, depending on length. This is more pronounced with higher dielectric constants, reaching nearly 20 percent for samples of D8623. The junctions present in other measurement methods might have a similar effect and would explain those data sheet values, which are too low.

Since nearly all of the loss tangents should be lower than the measured values, the data are obviously not usable. However, they are a measure of the achieved accuracy in loss tangent and are surprisingly precise not only for this equipment, but also for all nonresonator methods. Furthermore these and the values for the magnetic permeability, which must be nearly equal to 1, provide experimental proof of the correctness of the evaluation method as well as the range of accuracies. Possible accuracy improvements by three orders of magnitude in transmission data and two orders of magnitude in geometrical tolerances support the simulation of Section V: loss tangents of low-loss materials can be determined to an accuracy of 10^{-6} to 10^{-7}. This makes resonator methods obsolete, since the method presented here has the additional advantages of being broadband and having better absolute accuracy.

VII. SUMMARY

A universal formalism, the L^3 method, is used to calculate the properties of the test line from measured transmission/reflection coefficients, and all formula derivations are made without approximations. Due to the rigorous treatment, accuracies of 0.1 percent are potentially achievable and are dependent only on geometrical tolerances, so that L^3 can be considered as a standard method. The geometries of samples for material parameter measurements can be individually optimized with regard to every parameter of interest. Low-loss as well as high-loss materials, anisotropic or off-diagonal tensor elements, and the properties of fluids and powders can be determined. The demonstrated example of a rectangular H_{10} waveguide is usable from 1 to 300 GHz. Other waveguides and fundamental modes may also be advantageous in certain applications, as well as in the optical region.

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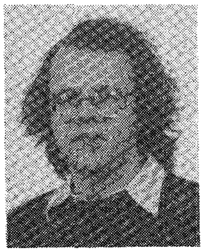

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